BELLCOMM, INC.
955 L'ENFANT PLAZA NORTH, S.W. WASHINGTON, D. C. 20024

B68 12067

SUBJECT: Additions to the MSFC Orbital Lifetime Program - Case 610

DATE: December 19, 1968

FROM: R. C. Purkey

ABSTRACT

Perturbations due to solar gravity, lunar gravity, and solar pressure have recently been added to the MSFC Orbital Lifetime Program. This MSFC developed perturbation subroutine did not affect the predicted lifetimes of the AAP mission, but would affect a satellite in a much higher orbit.

The principal equations and computational approach of the lifetime model are discussed. Basically, the program numerically integrates the secular derivatives of the orbital elements to calculate lifetime. Because the secular variations are slow, accuracy can be maintained even with relatively large integration steps. Thus, a 400 day lifetime can be computed in 1-1/2 minutes.

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MEMORANDUM FOR FILE

I. Introduction

Marshall Space Flight Center's Orbital Lifetime Program is the basis for the prediction of orbital lifetimes used in mission planning. Bellcomm's version of this program is now updated to include the effects of solar gravity, solar pressure, and lunar gravity in order to remain compatible with MSFC.

The program computes lifetimes by integrating the time derivatives of a set of orbital elements. This integration results in the knowledge of the satellite orbit elements but not the exact position of the satellite in orbit at a point in time. Since the orbital elements change very slowly until the last phase of the decay, integration steps of a few days may be used. These large integration steps result in very rapid computation. Even with this rapid computation, this method can calculate lifetimes of actual decayed satellites with an error as low as .25 percent.

II. The Orbital Lifetime Program

The orbital elements integrated by the program are the radius of perigee, radius of apogee, inclination, argument of perigee, and longitude of ascending node denoted by r_p, r_a, i, ω, and Ω respectively. The time rate of change of these elements is derived using the methods of general perturbation theory. Equations of the form

$$\frac{d\lambda}{dt} = f_R\{\lambda,t\}\bar{e}_R \cdot \bar{F} + f_L\{\lambda,t\}\bar{e}_L \cdot \bar{F} + f_H\{\lambda,t\}\bar{e}_H \cdot \bar{F} \qquad (1)$$

result for each orbital element, λ. In these equations $\bar{F}$ is the perturbation force and e_R, e_L, and e_H are unit vectors in an orthogonal system centered on the satellite.

The major perturbation force on an earth orbiting satellite is atmospheric drag. The drag force per unit mass is given by

$$\overline{F}_D = \frac{1}{2}\rho\overline{V}^2 \frac{C_D A}{m}$$

where: ρ = atmospheric density
$\overline{V}$ = relative velocity between satellite and atmosphere
C_D = drag coefficient
A = area normal to flow
m = mass of satellite.

Since drag depends on the relative velocity between the vehicle and the atmosphere, the inertial velocity of the satellite must be corrected for atmospheric rotation. The program assumes that the atmosphere is rigidly attached to the earth and therefore has an angular velocity equal to that of the earth. Only in-plane velocity components are considered resulting in the multiplication of the inertial velocity by the factor:

$$(1 - \frac{\omega_\rho}{n} \cos i)$$

where n is the mean motion of the satellite and ω_ρ is the rotation rate of the earth.

The area normal to the flow and the drag coefficients for the satellite are input data. There is a provision in the program for tumbling satellites in which the drag area and drag coefficient are corrected.

Atmospheric density is a function of altitude, position with respect to the earth-sun line, date, and time of day. Dependence of the density on time and date is due to variations of energy coming from the sun. This variation is taken into account in the program by relating it to the smoothed sunspot number, or the 10.7 cm flux. A table of these variables is published by the Space Environment Branch of MSFC every three months giving the predicted values of the 10.7 cm flux and smoothed sunspot number for the next 11 years. The latest updated tables should always be used in the program. These dynamic density relations are discussed in more detail in reference 2.

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The table of 10.7 cm flux and smoothed sunspot number gives values for a $+2\sigma$, nominal, and -2σ level of activity. Each level of solar activity produces a corresponding decay history, with the $+2\sigma$ level giving the shortest lifetime. MSFC has shown that this $+2\sigma$ level of activity will give a "guaranteed" lifetime. This "guaranteed" lifetime is considered to be the worst case and as such is used in mission planning work.

The accuracy of the lifetime model prediction is limited by the forecast of the solar activity input data. However, given good solar activity input data, the lifetime model is very accurate. For example, when MSFC used as input data the observed values of solar activity, predicted and actual satellite lifetimes agreed to within .25 percent.

The effect of a non-spherical earth can be incorporated into the density model by correcting the radius of the earth. This corrected earth radius is calculated by

$$R_e = R_{eq}\left[1 - f \sin^2 i \sin^2 (\omega+\Theta)\right] \qquad (2)$$

where: R_e = earths corrected radius
R_{eq} = earth radius at equator
f = flattening coefficient
Θ = true anomaly.

This value of the earth's radius is used to determine the altitude for the density table.

The earth's non-spherical gravitational field causes a small periodic oscillation of the satellite's radius. This causes the satellite's radius to be approximated by

$$r = \frac{a(1-e^2)}{1+e \cos\Theta} + \frac{J_2 R_{eq}^2}{a(1-e^2)}\left[\sin^2 i\left(1 - \frac{\sin^2(\omega+\Theta)}{2}\right) - \frac{1}{2}\right] \qquad (3)$$

where: J_2 = constant of earth's second zonal harmonic
e = eccentricity.

This oscillating radius is also used to determine the altitude for the density table.

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where: J_2 = constant of earth's second zonal harmonic
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This oscillating radius is also used to determine the altitude for the density table.

In-plane components of drag produce a perturbation only on the radius of perigee and radius of apogee. The time derivative of these elements are derived from Equation 1 by using drag as the perturbation force. In order to eliminate the periodic variations of these orbital elements and retain only the secular changes, these rates are averaged over one orbit. This results in

$$\dot{r}_a = \frac{-C_D A(1+e)^2\sqrt{\mu a}(1 - \frac{\omega_\rho}{n} \cos i)^2}{m(2\pi)} \int_0^{2\pi} \frac{\rho(1+2e \cos\theta + e^2)^{\frac{1}{2}}(1+\cos\theta)d\theta}{(1+e \cos\theta)^2} \tag{4}$$

$$\dot{r}_p = \frac{-C_D A(1-e)^2\sqrt{\mu a}(1 - \frac{\omega_\rho}{n} \cos i)^2}{m(2\pi)} \int_0^{2\pi} \frac{\rho(1+2e \cos\theta + e^2)^{\frac{1}{2}}(1-\cos\theta)d\theta}{(1+e \cos\theta)^2} \tag{5}$$

where: A = area used in drag force calculation
a = semi major axis
μ = gravitational constant for the earth.

These equations are seen to be integral equations over true anomaly and are integrated using Simpson's Rule in steps of 10°. In this integration the orbit elements are assumed constant. This results in the desired average rates.

The non-spherical gravitational field of the earth does produce secular variations in the longitude of ascending node and the argument of perigee. An analytic expression for this secular variation can be derived directly from the theory. Therefore no averaging need be made. This secular variation causes a rotation of the line of apsides and a regression of the line of nodes. The rate of change for these elements is given by:

$$\dot{\Omega} = - \frac{3}{2} \sqrt{\frac{\mu}{a^3}}\, J_2 \left(\frac{R_{eq}}{P}\right)^2 \cos i \tag{6}$$

$$\dot{\omega} = \frac{3}{4} \sqrt{\frac{\mu}{a^3}}\, J_2 \left(\frac{R_{eq}}{P}\right)^2 (4-5\sin^2 i) \tag{7}$$

where: $P = a(1-e^2)$.

Therefore Equations 4, 5, 6, and 7 represent the average decay rates. The radius of apogee and perigee rates depend only on the drag perturbation force with the effects of oblatness included in the density model. The rates of change of the longitude of ascending node and argument of perigee depend only on the non-spherical gravitational field. Inclination is not affected by these perturbations.

These average rates are then integrated over many orbits using a standard Runge-Kutta fourth order scheme. Either time or apogee radius may be used as the independent variable in the Runge-Kutta integration. In either case the integration step size may also be as a function of time or apogee radius respectively.

III. Solar and Lunar Effects

Perturbations due to solar pressure, solar gravity, and lunar gravity have been added to the Orbital Lifetime Program. The rate of change of each orbital element due to these perturbations is calculated separately and then added to the rates of change caused by the other forces.

Gravitational force perturbations are computed by

$$\bar{F}_{SG} = -\mu_S \left[\frac{\bar{r}_{SP}}{r_{SP}^3} - \frac{\bar{r}_{SE}}{r_{SE}^3} \right] \tag{8}$$

$$\bar{F}_{LG} = -\mu_M \left[\frac{\bar{r}_{MP}}{r_{MP}^3} - \frac{\bar{r}_{ME}}{r_{ME}^3} \right] \tag{9}$$

where: $\bar{r}_{SP}$ = vector from sun to satellite
$\bar{r}_{SE}$ = vector from sun to earth
$\bar{r}_{MP}$ = vector from moon to satellite
$\bar{r}_{ME}$ = vector from moon to earth.

These gravitation force perturbations are then substituted into Equation 1 to obtain the rates of change of the orbital elements. In this case all five orbital element rates become integral equations over 360° of true anomaly in order to eliminate the periodic terms.

The average rate of change of each element for this one orbit is again determined by using a Simpson's Rule integration in steps of 10° of true anomaly. Since the position vectors in Equations 8 and 9 are also functions of true anomaly, they must also be evaluated at each step. These average rates are then added to those caused by drag and oblatness before the Runge-Kutta integration.

Solar pressure will affect only those satellites with a large area to mass ratio. The perturbation force for this effect is computed by

$$\bar{F}_{SP} = -\frac{A}{m} P_r \bar{e}_{PS}$$

where: $\bar{e}_{PS}$ = unit vector from the satellite to the sun
P_r = solar pressure constant = 95.77 Newton/(meter)^2
A = area normal to sun's rays
m = mass of satellite.

This effect assumes that the direction from the earth to the sun is equal to the direction of the satellite to the sun. The same method as above is again employed to integrate these equations to obtain average rates of change for all five of the orbital elements.

These solar and lunar perturbations require a knowledge of the position of the sun and the moon. This data is supplied by a modified version of subroutine EPHEM from the BCMASP program. Since these effects are minor, the program uses the reference coordinates of the ephemeris tape and does <u>not</u> transform to a true inertial coordinate system of date.

Ephemeris tapes themselves seem to vary greatly. At present Bellcomm is using a JPL ephemeris tape which references to 1950 mean coordinates. Great care should be exercised when using or comparing the results of the various ephemeris tapes and reference coordinates. The use of a tape with different reference coordinates normally would not result in different lifetimes although the rates of decay at a given time may vary.

IV. Results of the Solar and Lunar Perturbations

In order to determine the effects of these additional perturbations, several test cases using the AAP mission in various orbits were analyzed. This study concluded that for orbits below 350 nm there were no significant differences caused by the perturbations of the sun or moon.

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A satellite of 25 kilogram mass and .05 square meter area was then analyzed in a 400 nm circular orbit. For this case the decay rates of apogee and perigee (the program defines an apogee and a perigee in the case of a circular orbit) are shown in Figure 1. These rates are for the case where atmospheric density is equal to zero and only solar and lunar gravitational perturbations are considered. Figure 1 shows that these effects cause the apogee and perigee rates to have a 180° phase difference. The magnitude of the rates at a given time is not equal, however. If the drag force were included in the rate calculations, this combination of forces could cause a difference in lifetimes.

V. Use of Program

The program is kept on tape and a Fastrand file for ready access. The computer time for only the drag perturbation lifetime, averages about 1-1/2 minutes for a 400-day lifetime. If all perturbations are included about 2-1/2 minutes are needed. If a study were to be undertaken using the additional perturbations, it is advisable to generate a Fastrand file with the ephemeris data needed. This would eliminate the lost computer time during multiple runs when a tape rewind and new search are executed. Since the Fastrand rewind is almost instantaneous and a smaller amount of data would be present to search, the lost time is reduced to less than 1/10 of that lost using tapes. Care must be taken to insure the ephemeris data on the file covers the dates of the lifetime.

VI. Summary

Solar and lunar perturbations have been added to the Orbital Lifetime Program to keep Bellcomm's version compatible with MSFC. These perturbations will affect satellites in a very high orbit or those with a very light weight and a large area. The AAP mission lifetime is unaffected by these additional perturbations.

R. C. Purkey

1025-RCP-dcs R. C. Purkey

BELLCOMM, INC.

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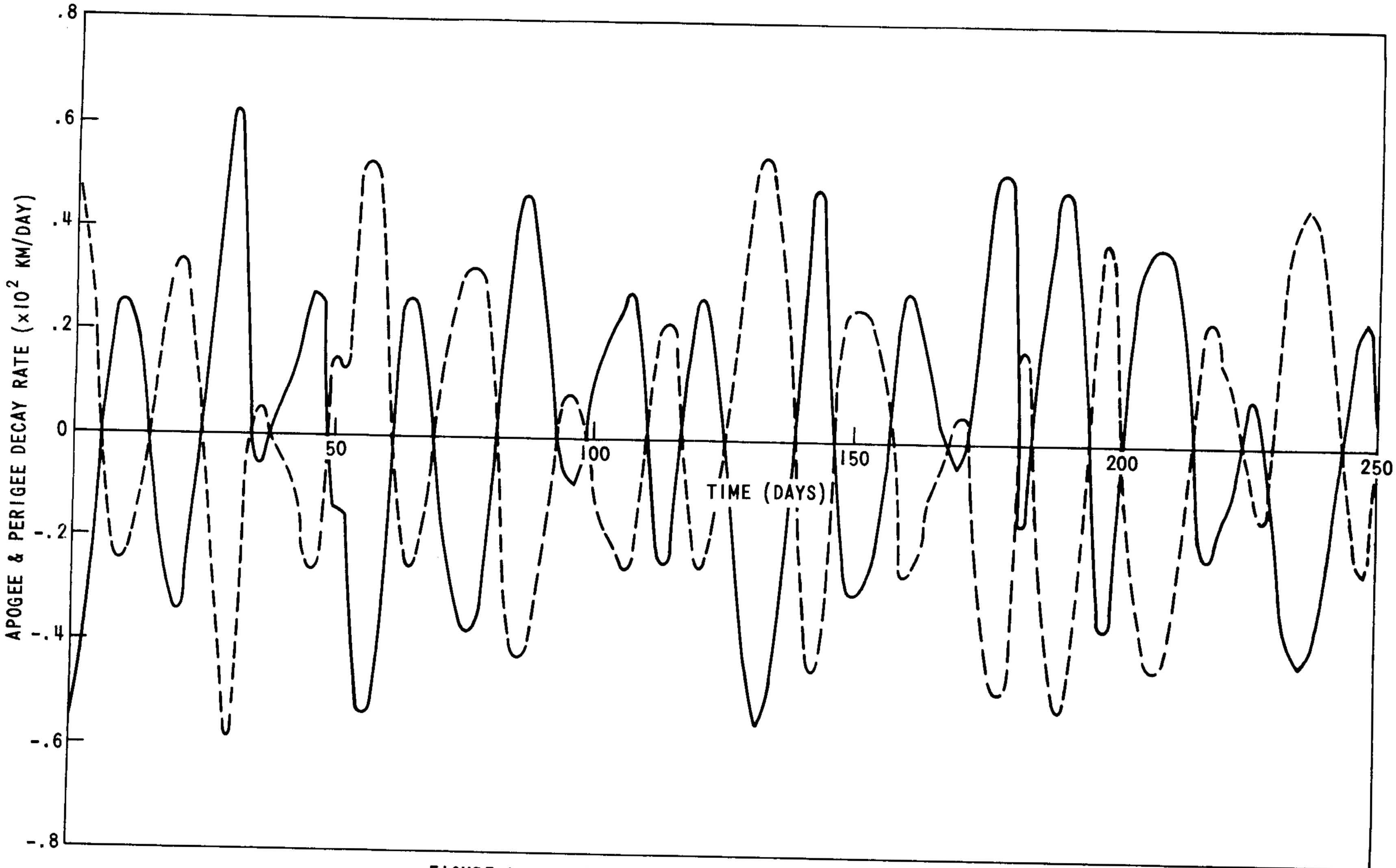

FIGURE I - DECAY RATES OF RADIUS OF APOGEE & PERIGEE

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